Solutions

1. (a)

	D	E	F
A	20	4	
B		26	6
C			14

M1

A1 2

(b) $S_A = 0$ $S_B = –1$ $S_C = 7$ M1
$D_P = 21$ $D_E = 24$ $D_F = 18$ A1

$I_{13} = I_{AF} = 16 – 0 – 18 = –2$
$I_{21} = I_{BD} = 18 + 1 – 21 = –2$ M1
$I_{31} = I_{CD} = 15 – 7 – 21 = –13$ (*) A1ft
$I_{32} = I_{CE} = 19 – 7 – 24 = – 12$ A1ft 5

(c) eg $CD(+) \to AD(–) \to AE(+) \to BE(–) \to BF(+) \to CF(–)$ $\theta = 14$ M1 A1ft

	D	E	F
A	6	18	
B		12	20
C	14		

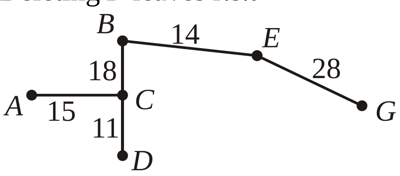
A1ft A1

cost £1384

4

[11]

2. (a) Deleting F leaves r.s.t



M1
r.s.t. length = <u>86</u> A1
s_0 lower bound = 86 + 16 + 19 = <u>121</u> M1 a1 4

$\therefore$ <u>best L.B is 129</u> by deleting C(ft from <u>choice</u>) B1 ft 1

(b) Add 33 to BF and FB B1
Add 31 to DE and ED B1 2

(c) Tour, visits each vertex, order correct using table of least distances. M1 A1
e.g. F C D A B E G F (actual route F C D C A B E G F)A1
upper bound of 138 km A1 4

[11]

3. Let x_{ij} be <u>number</u> of units transported from i to j

where $i \in \{W, X, Y\}$ and $j \in \{J, K, L\}$ B1 1

warehouse supermarket

<u>objective</u> minimise "C" $= 3x_{\mathrm{WJ}} + 6x_{\mathrm{WK}} + 3x_{\mathrm{WL}} +$ B1

$5x_{\mathrm{XJ}} + 8x_{\mathrm{XK}} + 4x_{\mathrm{XL}} +$ B1 2

$2x_{\mathrm{YJ}} + 5x_{\mathrm{YK}} + 7x_{\mathrm{YL}}$

subject to $x_{\mathrm{WJ}} + x_{\mathrm{WK}} + x_{\mathrm{UL}} = 34$ M1 A1

$x_{XJ} + x_{XK} + x_{XL} = 57$

$x_{\mathrm{YJ}} + x_{\mathrm{YK}} + x_{\mathrm{YL}} = 25$

$x_{\mathrm{WJ}} + x_{XY} + x_{\mathrm{YJ}} = 20$ A1 3

$x_{\mathrm{WK}} + x_{XK} + x_{\mathrm{YK}} = 56$

$x_{\mathrm{WL}} + x_{XL} + x_{\mathrm{YL}} = 40$

$x_{ij} \geq 0 \quad \forall \ i \in \{W, X, Y\}$ and $j \in \{J, K, L\}$ B1 1

[7]

4. (a) The route from start to finish in which the arc of minimum length is as large as possible. B2, 1, 0

e.g. must be pratical, involve choice of route, have are 'cuts'. B1 3

(b)

Stage	State	Action	Value		
1	H	HK	18(*)	M1 A1	2
	I	IK	19(*)		
	J	JK	21(*)		
2	F	FH	min (16,18) = 16		
		FI	min (23,19) = 19(*)	M1 A1 A1	3
		FJ	min (17,21) = 17		
	G	GH	min (20,18) = 18		
		GI	min (15,19) = 15		
		GJ	min (28,21) = 21(*)		
3	B	BG	min (18,21) = 18(*)		
	C	CF	min (25,19) = 19(*)	M1 A1ft	
		CG	min (16,21) = 16		
	D	DF	min (22,19) = 19(*)		
		DG	min (19,21) = 19(*)		
	E	EF	min(14,19) = 14(*)		
4	A	AB	min (24,18) = 18	A1ft	3
		AC	min (25,19) = 19(*)		
		AD	min (27,19) = 19(*)		
		AE	min (23,14) = 14		

(c) Routes $A \ C \ F \ I \ K$, $A \ D \ F \ I \ K$, $A \ D \ G \ J \ K$ A1ft A1ft A1ft 3

[14]

5. (a) To maximise, subtract all entries from $n \geq 30$ — M1

e.g. $\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & 5 & 3 & 6 \\ 0 & 3 & 5 & 9 \end{bmatrix}$ — A2,1,0 — 3

minimum uncovered element is 1: so $\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 6 & 3 & 0 \\ 0 & 4 & 2 & 5 \\ 0 & 2 & 4 & 8 \end{bmatrix}$ — M1 A2ft1ft0 — 3

<u>or</u> — M1

min. el. = 2 $\begin{bmatrix} 7 & 0 & 0 & 2 \\ 0 & 4 & 1 & 0 \\ 0 & 2 & 0 & 5 \\ 0 & 0 & 2 & 8 \end{bmatrix}$ min. el = 2 $\begin{bmatrix} 7 & 0 & 0 & 0 \\ 2 & 6 & 3 & 0 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 2 & 6 \end{bmatrix}$ — A2ft1ft0 — 3

$A - 2 \quad B - 4 \quad C - 3 \quad D - 1$ — M1 A1ft — 2
$A - 3 \quad B - 4 \quad C - 1 \quad D - 2$

(b) £1160 000 — B2,1,0 — 2

(c) Gives other solution — M1 A1ft — 2

[15]

6. (a) $SS_1 - 47$, $SS_2 - 87$, $T_1T - S_1$, $T_2T - T_3$ added to diagram 1 — M1 A1 — 2

If all 4 nos. zero then M0

M1 4 arcs added correctly + 4 numbers given (diagram 1 only) condone lack of arrows

A1 c.a.o. (diagram 1 only) penalise arrow errors here

(b)

$SS_1 \to 0$, $SS_2 \to 38$, $T_1T \to 8$, $T_2T \to 20$ — M1 A1 — 2

$\leftarrow 47 \qquad \leftarrow 49 \qquad \leftarrow 43 \qquad \leftarrow 53$

M1 4 arcs, 2 numbers and 2 arrows $\leftrightarrows$ *per arc*

A1 c.a.o.

(c) e.g. $S\ S_2\ A\ D\ T_1\ T$ — 2
$S\ S_2\ C\ E\ T_2\ T$ — 1 — M1
$S\ S_2\ C\ E\ D\ T_2\ T$ — 10 — A4,3,2,1,0
$S\ S_2\ C\ E\ B\ D\ T_1\ T$ — 4
Maximum flow — 113 — B1 — 6

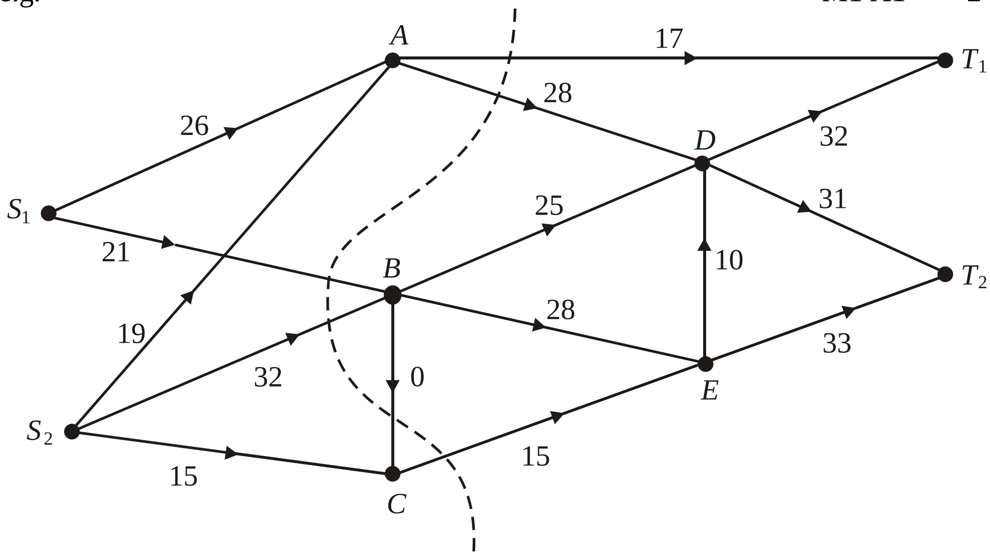
M1 2 correct routes + flows found (flow > 10 gets M0) (condone initial f.a. routes <u>only</u> if clearly repeated from new ones)

A4 all flows + routes to 15 more <u>or</u> flow increased above 17 more

A2 $\geq$ 3 flows + routes to 11 more <u>or</u>
A1 at least 2 flows + routes found to 5 more

B1 113 c.a.o.

(d) e.g. M1 A1 2



M1 consistent flow of 101(*), complete clear (doesn't need to ft from (c))

A1 correct flow of 113 including arrows

(e) Max flow – min cut theorem; cut AT_1, AD, S_1B, S_2B, BC, CE M1 A1 2

M1 flow of 113 + cut attempted + max flow – min cut theorem referred to (3 out of 4)

A1 c.a.o.

(f) Idea of a <u>directed</u> flow along arcs; from S to T; through a system/network; practical B2,1,0 2

B2 all 4 bits there

B1 2 out of 4 there

[16]

7. (a) A zero-sum game is one in which the sum of the gains for all players is zero. (o.e.) B1 1

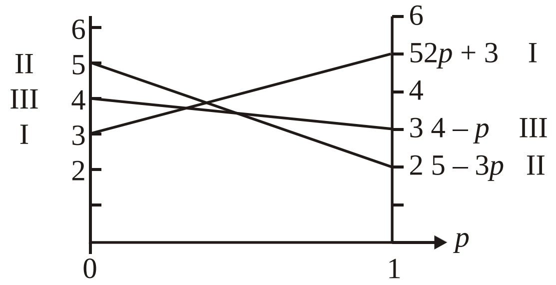
(b)

	I	II	III		
I	5	2	3	min 2	
II	3	5	4	min 3 ← max	M1 A1
	max 5	5	4 ↑ min		

Since 3 ≠ 4 not stable A1 3

(c) Let A play I with probability p
Let A play II with probability $(1 - p)$

If B plays I A's gains are $5p + 3(1 - p) = 2p + 3$
If B plays II A's gains are $2p + 5(1 - p) = 5 - 3p$ M1 A1 2
If B plays III A's gains are $3p + 4(1 - p) = 4 - p$



A2,1,0 2

Intersection of $2p + 3$ and $4 - p \Rightarrow p = \frac{1}{3}$ M1 A1ft 2

$\therefore$ A should play I $\frac{1}{3}$ of time and II $\frac{2}{3}$ of time; value (to A) = $3\frac{2}{3}$ A1ft A1ft 2

(d) Let B play I with probability q_1,
II with probability q_2 and
III with probability q_3 B1

e.g. Let $x_1 = \frac{q_1}{v}$ $x_2 = \frac{q_2}{v}$ $x_3 = \frac{q_3}{v}$ M1

Maximise P = $x_1 + x_2 = x_3$ A1

subject to $5x_1 + 2x_2 + 3x_3 \leq 1$
$3x_1 + 5x_2 + 4x_3 \leq 1$ A2,1,0 5

$x_1, x_2, x_3 \geq 0$

[17]

Alt 1

e.g. $\begin{bmatrix} -5 & -3 \\ -2 & -5 \\ -3 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 \\ 4 & 1 \\ 3 & 2 \end{bmatrix}$

maximise P = V

subject to $v - q_1 - 4q_2 - 3q_3 \leq 0$
$v - 3q_1 - q_2 - 2q_3 \leq 0$ $q_1 + q_2 + q_3 \leq 1$
$v, q_1, q_2, q_3 \geq 0$ or = 1

8. (a) r, s and t are unused amounts of bird seed (in kg), suet blocks and peanuts (in kg) that Polly has at the end of each week after shehas made up and sold her packs B2,1,0 2

B2 Ref to "unused" "bird seed, suet blocks & peanuts"

B1 Ref to "unused" <u>*or*</u> *bird seed etc* <u>*or*</u> *muddled explanation.*

"bad" gets B1 must engage with context

(b)

b.v.	x	y	z	r	s	t	value		
z	$\frac{2}{5}$	$\frac{1}{2}$	1	$\frac{1}{10}$	0	0	14	$R_1 \div 10$	M1 A1
s	$\frac{2}{5}$	–1	0	$-\frac{2}{5}$	1	0	4	$R_2 - 4R_1$	M1

t	$-\frac{1}{5}$	$\frac{1}{2}$	0	$-\frac{3}{10}$	0	1	18	$R_3 - 3R_1$	A2ft, 1ft, 0	5
p	–90	–25	0	65	0	0	9100	$R_4 + 650R_1$		

M1 correct pivot

A1 pivot row correct c.a.o. incl.bv

M1ft correct row operations used (all 3) – at least 1 non zero or 1 term correct in each row.
Where row not ft ⇒ M0

A2ft non-pivoted rows correct; –1 each error ft on error in pivot choice only.
Penalise b.v once only

(c) $x = 0$ $y = 0$ $z = 14$ $r = 0$ $s = 4$ $t = 18$ $p = £91$ — M1 A2ft, 1ft, 0 — 3

M1 3 variables stated – must have completed b.v. + value columns on tableau.
Any negatives M0

A1ft all 7 c.a.o. Need £91 ft but accept 9100

A1ft at least 4 c.a.o. (condone P = 9100ft)

(d) $p - 90x - 2\sqrt{y} + 65r = 9100$ (o.e.) — M1 A1ft — 2

M1ft P, (–)90x, (–)25y, 65r and 9100 (or 91) all present and one = sign

A1ft c.a.o. (o.e.)

(e) $p = 9100 + 90x + 25y - 65r$
So increasing x or y would increase the profit — B1ft — 3

B1ft stating that increasing x or y would increase profit, probably re-arranging profit equation. Generous.

(f) The $\frac{2}{5}$ in the x column and 2nd (s) row. — B2ft, 1ft, 0 — 2

B2ft $\frac{2}{5}$ identified, x column and 2nd (s) row.

Accept ringed in last tableau

B1ft "bad" gets B1, if ft their "optional" tableau B1.

[15]

(b) *Notes*

1. Wrong pivot chosen in col 2 (–usually 4) M0 then for M1A2ft

(a)

b.v.	x	y	z	r	s	t	value	
r	–1	$2\frac{1}{2}$	0	1	$-2\frac{1}{2}$	0	–10	$R_1 - 10R_2$
z	$\frac{1}{2}$	$\frac{1}{4}$	1	0	$\frac{1}{4}$	0	15	$R_2 \div 4$
t	$-\frac{1}{2}$	$1\frac{1}{4}$	0	0	$-\frac{3}{4}$	1	15	$R_3 - 3R_2$
p	–25	$-187\frac{1}{2}$	0	0	$162\frac{1}{2}$	0	9750	$R_4 + 650R_2$

(b)

b.v.	x	y	z	r	s	t	value	
r	$\frac{2}{3}$	$-1\frac{2}{3}$	0	1	0	$\frac{-10}{3}$	–60	$R_1 - 10R_3$
s	$\frac{2}{3}$	$-1\frac{2}{3}$	0	0	1	$\frac{-4}{3}$	–20	$R_2 - 4R_3$
z	$\frac{1}{3}$	$\frac{2}{3}$	1	0	0	$\frac{1}{3}$	20	$R_3 \div 3$
p	$-133\frac{1}{3}$	$83\frac{1}{3}$	0	0	0	$216\frac{2}{3}$	13000	$R_4 + 650R_3$

2. *MISREADS* – use col x or col y (–2 A marks if earned)

(a)

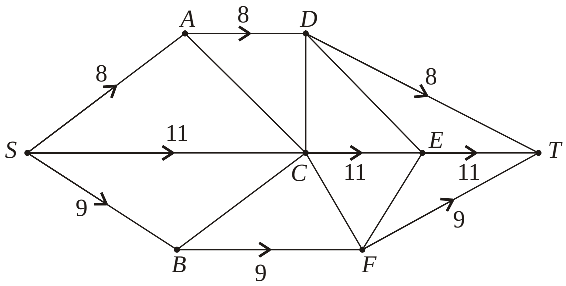
b.v.	x	y	z	r	s	t	value	
r	0	3	2	1	–2	0	20	$R_1 - 4R_2$
x	1	$\frac{1}{2}$	2	0	$\frac{1}{2}$	0	30	$R_2 \div 2$
t	0	$1\frac{1}{2}$	1	0	$-\frac{1}{2}$	1	30	$R_3 - R_2$
p	0	–175	50	0	175	0	10500	$R_4 + 350R_2$

(b)

b.v.	x	y	z	r	s	t	value	
y	$\frac{4}{5}$	1	2	$\frac{1}{5}$	0	0	28	$R_1 - 5$
s	$1\frac{1}{5}$	0	2	$-\frac{1}{5}$	1	0	32	$R_2 - R_1$
t	$-\frac{3}{5}$	0	–1	$-\frac{2}{5}$	0	1	4	$R_3 - 2R_1$
p	–70	0	50	70	0	0	9800	$R_4 + 350R_2$

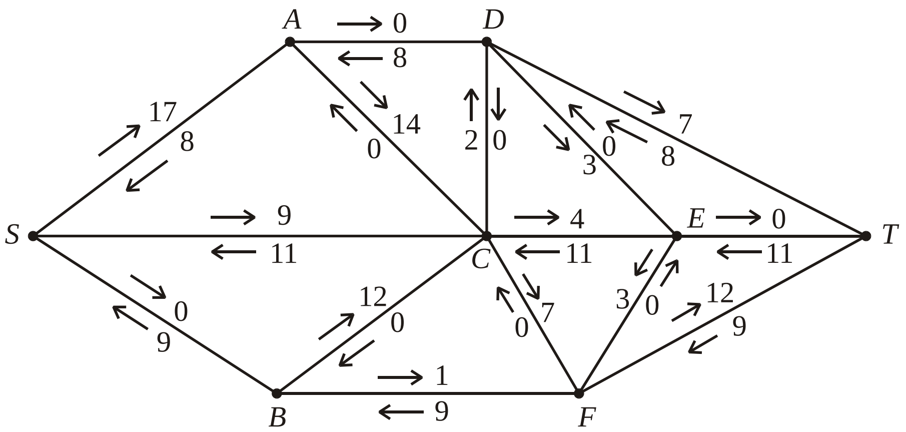
9. (a) SADT – 8 SCET – 11 SBFT – 9 B2, 1, 0

(b)



B1 3

(c) (i)



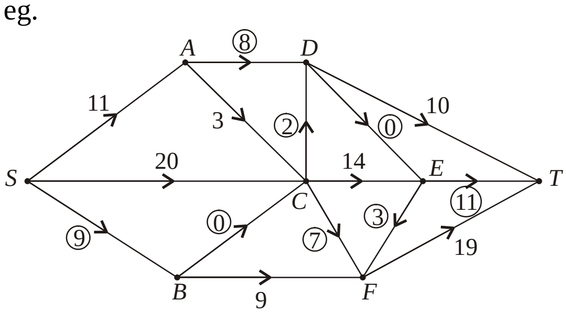
M1
A1 2

e.g.

S A C D T – 2 S C F T – 6 A1
S A C E F T – 3 S A C F T – 1 A1 3

max flow 40

(ii) eg.



M1 A1 2

(iii) Max flow – min cut theorem M1
cut AD, CD, DE, ET, CF, BC, SB ie {S A C E } {B D F T} A2, 0 3

(d) Idea of a directed flow through a system of arcs from S to T practical B1 1

[14]